ON GECKELER'S EQUATIONS IN THE THEORY OF GYROCOMPASSES

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In this work we investigate natural motions of a twin gyrocompass which does not possess properties of a space gyrocompass.

Within the limits of the precessional theory we are investigating the validity of the simplified theory of a gyrocompass which has been introduced by Geckeler.

We investigate the stability of the trivial solution of the equations with variations for the regular circulation of a ship. We have obtained explicit expressions for the characteristic indices of the system of equations with periodic coefficients, describing the motion of the sensing element of a gyrocompass during a circulation.

1. The equations with variations characterizing the motion of the sensing element of a twin gyrocompass of the Anschütz type, in which the forced ballistic deviations are fully compensated, have the form [2]

$$\frac{Pl}{g}(V\alpha)^{\cdot} - Pl\beta - 2B\sin\varepsilon_0\Omega\delta = 0, \qquad \gamma^{\cdot} + \frac{s}{2B\sin\varepsilon_0}\delta + \Omega\beta = 0$$
$$\beta^{\cdot} + \frac{V\alpha}{R} - \Omega\gamma = 0, \qquad (2B\sin\varepsilon_0\delta)^{\cdot} - Pl\gamma + \frac{Pl}{g}\Omega V\alpha = 0 \qquad (1.1)$$

The notation is the same as in the paper [2]: a dot denotes the time drivative.

If we neglect in (1.1) terms containing the angular velocity Ω as a factor (more exactly, the z° component of the angular velocity vector, where z° is the vertical axis of the triad $Ox^\circ y^\circ z^\circ$ oriented along the vector V of the velocity of the suspension point [1 and 2]) then the system (1.1) can be broken into two independent systems of the form

$$\frac{Pl}{g}(V\alpha)^{\cdot} - Pl\beta = 0, \qquad \gamma^{\cdot} + \frac{s}{2B\sin\varepsilon_0}\delta = 0$$
$$\beta^{\cdot} + \frac{V\alpha}{R} = 0, \qquad (2B\sin\varepsilon_0\delta)^{\cdot} - Pl\gamma = 0 \qquad (1.2)$$

Systems of the form (1.2) were investigated first by Geckeler in the papers [3 and 4] known to the experts in the theory and applications of gyroscopes. Geckeler failed failed to give a rigorous derivation of conditions under which the simplified system (1.2) is permissible.

For a space gyrocompass [1] it has been shown in [5] that the equations of motion of its base on the Earth's sphere can be reduced to (1.2). A similar investigation for a gyrocompass which does not possess properties of a space gyrocompass has never been carried out, as far as we know, and for this reason we make here a special study on the applicability of the system (1.2).

2. Further on we shall need certain auxiliary systems, arising from (1.1) and (1.2). Substituting in (1.1)

$$V\alpha = x_1, \qquad 2B \sin \varepsilon_0 g\delta \,/\, Pl = x_4 \tag{2.1}$$

and denoting β and γ, respectively, by x_2 and x_3 we obtain

$$\begin{matrix} x_1^{\cdot} - gx_2 - \Omega x_4 = 0, & x_3^{\cdot} + \dfrac{p^2}{g} x_4 + \Omega x_2 = 0 \\ x_2^{\cdot} + \dfrac{x_1}{R} - \Omega x_3 = 0, & x_4^{\cdot} - gx_3 + \Omega x_1 = 0 \end{matrix} \qquad \left(p = \frac{\sqrt{Pls}}{2B\sin\varepsilon_0}\right) \tag{2.2}$$

From (1.2) we have,respectively

$$x_1^{\cdot} - gx_2 = 0, \quad x_3^{\cdot} + \frac{p^2}{g} x_4 = 0, \quad x_2^{\cdot} + \frac{x_1}{R} = 0, \quad x_4^{\cdot} - gx_3 = 0 \tag{2.3}$$

The roots of the equations of the independent systems (2.3) are

$$r_{1,2} = \pm \nu i, \qquad r_{3,4} = \pm pi \qquad (\nu = \sqrt{g/R}) \tag{2.4}$$

and they correspond to the undamped vibrations of a gyrocompass with the frequencies ν and p. From (2.2) we have also

$$\begin{matrix} x_1^{\cdot\cdot} + (\nu^2 - \Omega^2)\, x_1 - 2\Omega x_4^{\cdot} - \Omega^{\cdot} x_4 = 0 \\ x_4^{\cdot\cdot} + (p^2 - \Omega^2)\, x_4 + 2\Omega x_1^{\cdot} + \Omega^{\cdot} x_1 = 0 \end{matrix} \tag{2.5}$$

In some cases it is advisible to introduce the variables x_1 and x_4 in another way, assuming, for example

$$V\alpha = Ru\cos\varphi x_1, \qquad \delta = x_4 \sin\varphi \,/\, \sin\varepsilon_0 \tag{2.6}$$

If the parameters of a compass are subjected to the condition

$$2Bg = PlRu \tag{2.7}$$

and if we set φ = const , then we obtain from (1.1) [2]

$$\begin{matrix} x_1^{\cdot} - \dfrac{\nu^2}{u\cos\varphi} x_2 - \Omega \tan\varphi x_4 = 0, & x_3^{\cdot} + \dfrac{p^2}{\nu^2} u \sin\varphi x_4 + \Omega x_2 = 0 \\ x_2^{\cdot} + u\cos\varphi x_1 - \Omega x_3 = 0, & x_4^{\cdot} - \dfrac{\nu^2}{u\sin\varphi} x_3 + \Omega \cot\varphi x_1 = 0 \end{matrix} \tag{2.8}$$

Regarding Ω as a given function of t, we can pass to the variables ξ_1 and ξ_2 in (2.5) if we substitute [8] (*)

*) Let us mention that a transformation of the form (2.9) has been previously used in the works [6 and 7] which do not deal strictly with the theory of a gyrocompass. In the papers [5 and 8] on the theory of a gyrocompass there are no references to those earlier sources.

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$$\xi_1 = x_1 \cos\theta - x_4 \sin\theta \qquad \left(\theta = \int_0^t \Omega(\tau)\, d\tau\right) \tag{2.9}$$
$$\xi_2 = x_1 \sin\theta + x_4 \cos\theta$$

From (2.9) we have the reciprocal relations

$$x_1 = \xi_1 \cos\theta + \xi_2 \sin\theta, \qquad x_4 = -\xi_1 \sin\theta + \xi_2 \cos\theta \tag{2.10}$$

from which we obtain the equations for ξ_1 and ξ_2 in the form

$$\begin{aligned} &\xi_1^{\cdot\cdot} + {}^1/_2\,[p^2 + \nu^2 - (p^2 - \nu^2)\cos 2\theta\,]\,\xi_1 - {}^1/_2\,(p^2 - \nu^2)\sin 2\theta\,\xi_2 = 0 \\ &\xi_2^{\cdot\cdot} + {}^1/_2\,[p^2 + \nu^2 + (p^2 - \nu^2)\cos 2\theta\,]\,\xi_2 - {}^1/_2\,(p^2 - \nu^2)\sin 2\theta\,\xi_1 = 0 \end{aligned} \tag{2.11}$$

Together with the system (2.5) we shall consider the system [7]

$$\begin{aligned} &x_1^{\cdot\cdot} + 2bx_1^{\cdot} + (\nu^2 - \Omega^2)\,x_1 - 2\Omega x_4^{\cdot} - \Omega^{\cdot} x_4 = 0 \\ &x_4^{\cdot\cdot} + 2bx_4^{\cdot} + (p^2 - \Omega^2)\,x_4 + 2\Omega x_1^{\cdot} + \Omega^{\cdot} x_1 = 0 \end{aligned} \tag{2.12}$$

containing arbitrarily small dissipative terms.

Using the transformation (2.9) we obtain

$$\begin{aligned} &\xi_1^{\cdot\cdot} + 2b\xi_1^{\cdot} + {}^1/_2\,[p^2 + \nu^2 - (p^2 - \nu^2)\cos 2\theta\,]\,\xi_1 + \\ &\qquad + [2b\Omega - {}^1/_2\,(p^2 - \nu^2)\sin 2\theta\,]\,\xi_2 = 0 \\ &\xi_2^{\cdot\cdot} + 2b\xi_2^{\cdot} + {}^1/_2\,[p^2 + \nu^2 + (p^2 - \nu^2)\cos 2\theta\,]\,\xi_2 - \\ &\qquad - [2b\Omega + {}^1/_2\,(p^2 - \nu^2)\sin 2\theta\,]\,\xi_1 = 0 \end{aligned} \tag{2.13}$$

We shall dwell briefly on these cases of the motion of the base when the solution of the problem does not present special difficulties.

It occurs in all cases when the quantity Ω can be regarded constant, i.e. when the gyrocompass works on a base fixed with respect to the Earth's surface, and also when the motion is along a parallel of latitude with a constant eastern component v_E of its velocity. In the first case $\Omega = u\sin\varphi$, in the second case we set $\Omega = u\sin\varphi\,(1 + v_E/Ru\cos\varphi)$. In these cases Equations (2.2) can be integrated in a closed form, consequently we can easily find out the influence of the terms containing Ω as a factor.

When a ship moves arbitrarily in a straight course line with constant velocity the latitude of its position varies as a rule very slowly. Consequently we can neglect its rate of change and at every given latitude we can assume that Ω = const . We shall not analyse these cases [11]. We shall only show that under these special conditions the motion of a gyrocompass which does

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The transformation (2.9) leads to systems with constant coefficients, of the form of the equations of motion of the Schuler pendulum, which control the motion of a gyrocompass possessing the propeties of the Geckeler-Ishlinskii space gyrocompass. This problem is dealt with in the paper [5] in which, by using Liapunov's theorem on the applicability of the equations with periodic coefficients to the equations with constant coefficients, are derived explicit expressions for the four first integrals of the initial system, which determine the form of the nonsingular transformation (2.9). The equations of motion of a gyrocompass which does not possess properties of a space gyroscompass, do not have those integrals, hence in this case the Ishlinskii-Andreev transformation, applicable to systems with constant coefficients, cannot be used. In a more general case the problem was investigated by Liashenko [9] who used the theorem of N.P. Erugin. The work of Andreev [10] does contain not very exact information on the essence of the problem.

not possess the properties of a space gyrocompass, can be described with sufficient, for all practical purposes, accuracy by systems of equations of the Geckeler type (1.2).

3. Let a ship describe a regular circulation with a constant velocity v and from a certain initial course ψ_0. The northern and the eastern components of velocity of the ship vary, respectively, as follows

$$v_N = v\cos(\psi_0 \pm \omega t), \qquad v_E = v\sin(\psi_0 \pm \omega t) \tag{3.1}$$

where ω is the angular frequency of the circulation, the upper signs refer to the right circulation, the lower to the left one. Considering the left circulation and keeping in the expression for Ω only the primcipal term which takes care of the rate of change of the course-line deviation, we shall assume as in [2]

$$\Omega = \frac{v_N}{Ru\cos\varphi} = \mu\omega\sin(\psi_0 - \omega t) \qquad \left(\mu = \frac{v}{Ru\cos\varphi}\right) \tag{3.2}$$

Consequently, by (2.9) we have

$$\theta = \mu\,[\cos(\psi_0 - \omega t) - \cos\psi_0] \tag{3.3}$$

Since the nonsingular transformation (2.9) which reduces the system (2.12) to (2.13), possesses in the case of a circulation of a ship periodic coefficients with the period $2\pi/\omega$, the characteristic indices can be derived from the system (2.13).

To obtain explicit expressions for the characteristic indices we shall apply to (2.13) the method of averaging [12] replacing periodic coefficients of this system by their average values for one period of circulation.

When averaging the functions $\sin 2\theta$ and $\cos 2\theta$, where $\theta(t)$ is given by (3.3), it is convenient to use the following expansions

$$\begin{aligned}\sin(2\mu\cos x) &= 2\sum_{n=0}^{\infty}(-1)^n J_{2n+1}(2\mu)\cos[(2n+1)x]\\ \cos(2\mu\cos x) &= J_0(2\mu) + 2\sum_{n=1}^{\infty}(-1)^n J_{2n}(2\mu)\cos(2nx)\end{aligned} \tag{3.4}$$

where $J_\nu(2\mu)$ is a Bessel function of an integral, nonnegative index.

As a result of averaging the system (2.13) becomes

$$\begin{aligned}&\xi_1^{\cdot\cdot} + 2b\xi_1^{\cdot} + {}^1/_2\,[p^2 + \nu^2 - (p^2 - \nu^2)\,J_0(2\mu)\cos(2\mu\cos\psi_0)]\,\xi_1 +\\ &\qquad + {}^1/_2\,(p^2 - \nu^2)\,J_0(2\mu)\sin(2\mu\cos\psi_0)\,\xi_2 = 0\\ &\xi_2^{\cdot\cdot} + 2b\xi_2^{\cdot} + {}^1/_2\,[p^2 + \nu^2 + (p^2 - \nu^2)\,J_0(2\mu)\cos(2\mu\cos\psi_0)]\,\xi_2 +\\ &\qquad + {}^1/_2\,(p^2 - \nu^2)\,J_0(2\mu)\sin(2\mu\cos\psi_0)\,\xi_1 = 0\end{aligned} \tag{3.5}$$

Introducing in (3.5) the new variables u_1 and u_2 given by Formulas

$$\begin{aligned}\xi_1 &= u_1\cos(\mu\cos\psi_0) + u_2\sin(\mu\cos\psi_0)\\ \xi_2 &= -u_1\sin(\mu\cos\psi_0) + u_2\cos(\mu\cos\psi_0)\end{aligned} \tag{3.6}$$

we obtain for u_1 and u_2 two independent equations

$$u_1^{\cdot\cdot} + 2bu_1^{\cdot} + k_1^2u_1 = 0, \qquad u_2^{\cdot\cdot} + 2bu_2^{\cdot} + k_2^2u_2 = 0 \tag{3.7}$$

where

$$k_1^2 = {}^1/_2\,[p^2 + \nu^2 - (p^2 - \nu^2)\,J_0\,(2\mu)]$$
$$k_2^2 = {}^1/_2\,[p^2 + \nu^2 + (p^2 - \nu^2)\,J_0\,(2\mu)] \tag{3.8}$$

The roots of the characteristic equations, corresponding to (3.7) have at $b \neq 0$ nonvanishing real parts, therefore the application of the method of averaging is legitimate [12]. Since $b > 0$, and k_1^2 and k_2^2 are always greater than zero, the gyrocompass is stable at a sufficiently small period of circulation T. In practice, this condition is as a rule satisfied, because T is always small as compared with the period of the natural oscillation of the compass T_0 [2 and 12].

In this way by introducing arbitrarily small dissipative forces we make a gyrocompass asymptotically stable. When damping is weak the characteristic indices $\varkappa_k$ of the system (2.13) approach the roots of the characteristic equations corresponding to (3.7) and we can set [12]

$$\varkappa_{1,2} = -b \pm k_1 i, \qquad \varkappa_{3,4} = -b \pm k_2 i \tag{3.9}$$

Without damping

$$\varkappa_{1,2} = \pm k_1 i, \qquad \varkappa_{3,4} = \pm k_2 i \tag{3.10}$$

In practice we have always $\mu < 1$. Taking this into account and keeping only the first two terms in the expansion of $J_0(2\mu)$ in powers of μ, we obtain from (3.8) an expression very convenient for calculating the squares of the frequencies

$$k_1^2 = \nu^2 + {}^1/_2\,(p^2 - \nu^2)\,\mu^2, \qquad k_2^2 = p^2 - {}^1/_2\,(p^2 - \nu^2)\,\mu^2 \tag{3.11}$$

To the frequencies k_1 and k_2 correspond, respectively, the periods T_1 and T_2 which are determined with accepted degree of accuracy by Formulas

$$T_1 = \frac{2\pi}{\nu}\left[1 - \frac{1}{4}\left(\frac{p^2}{\nu^2} - 1\right)\mu^2\right], \qquad T_2 = \frac{2\pi}{p}\left[1 + \frac{1}{4}\left(1 - \frac{\nu^2}{p^2}\right)\mu^2\right] \tag{3.12}$$

If the parameter μ is so small that we can set $J_0(2\mu) = 1$, then from (3.8) we obtain $k_1 = \nu$ and $k_2 = p$, hence and also from (3.10)

$$\varkappa_{1,2} = \pm \nu i, \qquad \varkappa_{3,4} = \pm p i \tag{3.13}$$

These values of the characteristic indices coincide with the expressions (2.4) for the roots of the characteristic equations, obtained from the system of Geckeler (1.2) and (2.3).

If the compass parameters are such that $p = \nu$, then from (3.8) follows that $k_1 = k_2 = \nu$, and then $\varkappa_{1,2} = \pm \nu i$, $\varkappa_{3,4} = \pm \nu i$. These expressions for the characteristic indices were obtained previously in [5 and 8].

4. Let us consider the example of a circulating ship when $v = 30$ knots, $T = 4$ min. and $\varphi = 80°$.

Taking the numerical values for the parameters of the sensing element as in [2] we have

$$\mu^2=3.69\cdot10^{-2}, \qquad p^2=2.03\cdot10^{-5}\ \text{sec}^{-2} \tag{4.1}$$

Setting $\nu^2 = 0.154 \times 10^{-5}\,\text{sec}^{-2}$, we obtain from (3.8) that

$$k_1=1.37\cdot10^{-3}\ \text{sec}^{-1},\ k_2 = 4.46\cdot10^{-3}\ \text{sec}^{-1}$$

Hence we have $T_1 = 76.5$ min., $T_2 = 23.4$ min.

The characteristic indices which correspond to the frequancies k_1 and k_2 are by (3.10)

$$\varkappa_{1,2} = \pm\, 1.37\cdot10^{-3}\, i, \qquad \varkappa_{3,4}= \pm\, 4.46\cdot10^{-3}\, i \tag{4.2}$$

We shall compare our calculations with the results from paper [2] where the characteristic equation is derived for the system with periodic coefficients (2.8) which for our case has the recursive form

$$\rho^4 + A_1\rho^3 + A_2\rho^2 + A_1\rho + 1 = 0 \tag{4.3}$$

This equation was also calculated on the high speed computer "STRELA" and for the above numerical data it has the form

$$\rho^4 - 2.848\rho^3 + 3.809\rho^2 - 2.848\rho + 1.000 = 0 \tag{4.4}$$

The roots of this equation are

$$\rho_{1,2} = 0.946 \pm 0.324\, i, \qquad \rho_{3,4}=0.479 \pm 0.880\, i \tag{4.5}$$

with moduli equalling unity.

The characteristic indices corresponding to these roots are determined from Formulas

$$\varkappa_k = \frac{1}{T} \ln \rho_k \tag{4.6}$$

and have the following numerical values:

$$\varkappa_{1,2} = \pm\, 1.37\cdot10^{-3}\, i, \qquad \varkappa_{3,4} = \pm\, 4.46\cdot10^{-3}\, i \tag{4.7}$$

which coincides with the results in (4.2) obtained by the method of averaging.

5. It is necessary to mention that a formal application of the method of averaging to the systems (2.5) or (2.12) can lead to wrong conclusions about their stability. Indeed, the system (2.12) is a apecial case of the system with variable gyroscopic and nonconservative forces of the form [13]

$$x_j^{\cdot\cdot} + 2bx_j^{\cdot} + \lambda_j(t)\, x_j + \sum_k \left[g_{jk}(t)\, x_k^{\cdot} + e_{jk}(t)\, x_k\right] = 0 \tag{5.1}$$

where

$$g_{jk} = -\, g_{kj}, \qquad e_{jk} = -\, e_{kj}, \qquad g_{jj} = e_{jj} = 0 \tag{5.2}$$

When a ship circulates these forces will have periodic coefficients of periods equalling the period of a circulation.

When we formally average the system (2.12), the coefficients of 2Ω and of $\Omega^{\cdot}$ corresponding to g_{jk} and e_{jk} in (5.1) vanish on the strength of (3.2). Therefore the averaged equations obtained from (2.12) are a special case of Equations

$$x_j^{\cdot\cdot} + 2bx_j^{\cdot} + \lambda_j^* x_j = 0 \tag{5.3}$$

where by λ_j^* we denote the averaged coefficents $\lambda_j(t)$.

From the investigations of the stability of Equations (5.3) no conclusions on the stability of the system (5.1) should be made, because in (5.3) we neglected the variable gyroscopic forces, which in our case stabilize the gyrosphere with respect to the coordinate γ. Indeed, an equation of the form (5.3) can be obtained from (2.12) for the variable x_1 if we set x_4

and $x_4^{\cdot}$ equal to zero. Mechanically this corresponds to the motion of a system when gyroscopes cannot precess about the axes of their casings through an angle δ . Thus by formally averaging the system (2.5) with respect to the period of circulation we are losing the main advantage of a twin-rotor gyroscope, which is the gyroscopic moment $\Gamma = 2B \sin c_0 \delta^{\cdot}$, stabilizing the gyroshere with respect to the line North – South.

Transformation (2.9), however, puts (2.5) directly in the form

$$\xi_j^{\cdot\cdot} + \sum_k c_{jk}(t)\,\xi_k = 0 \qquad (c_{jk} = c_{kj}) \tag{5.4}$$

which is being expressed by the equations (2.11).

The gyroscopic forces are thus taken into account; they are included in the transformation (2.9). Using this transformation and introducing at the same time arbitrarily small dissipative forces the method of averaging does not present any difficulties and facilitates considerably the process of finding characteristic indices $\varkappa_k$.

To obtain $\varkappa_k$ we could also use the method of small parameters [14]. It is convenient to use μ as the small parameter. Following the procedure shown in [14], we have to set $\varkappa_k = \varkappa_k(0) + f_k(\mu)$, where $\varkappa_k(0)$ are the values of the characteristic indices $\mu = 0$. We must mention, however, that the method proposed in [14] for finding the function $f_k(\mu)$ leads in our case to very lengthy calculations because in the expressions for $\varkappa_k$ we must take into account the terms with μ^2 (see Formulas (3.10) and (3.11)).

6. We shall consider now a circulation from the eastern course and try to obtain formulas describing the motion of the sensing element in the cooi dinates α, β, γ and δ . Substituting in (3.5) $\psi_0 = \frac{1}{2}\pi$ and neglecting damping we obtain

$$\xi_1 = \xi_1(0)\cos k_1 t + k^{-1}\xi_1^{\cdot}(0)\sin k_1 t \tag{6.1}$$

Further, from (2.9), (3.2) and (3.3) and also by (2.2) we have

$$\xi_1(0) = x_1(0), \qquad \xi_1^{\cdot}(0) = g x_2(0) \tag{6.2}$$

consequently

$$\xi_1 = x_1(0)\cos k_1 t + g k_1^{-1} x_2(0)\sin k_1 t \tag{6.3}$$

Similarly

$$\xi_2 = x_4(0)\cos k_2 t + g k_2^{-1} x_3(0)\sin k_2 t \tag{6.4}$$

Further, by (2.10) we obtain the expressions for x_1 and x_4, and then from (2.2) the expressions for x_2 and x_3 . Passing according to (2.1) to the initial variables, we obtain finally

$$\begin{aligned}\alpha = V^{-1}\{&(V(0)\,\alpha(0)\cos k_1 t + g k_1^{-1}\beta(0)\sin k_1 t)\cos\theta + \\ &+ (g k_2^{-1}\gamma(0)\sin k_2 t + 2Bg(Pl)^{-1}\sin\varepsilon_0(0)\,\delta(0)\cos k_2 t)\sin\theta\}\end{aligned} \tag{6.5}$$

$$\begin{aligned}\beta = &(-k_1 g^{-1} V(0)\,\alpha(0)\sin k_1 t + \beta(0)\cos k_1 t)\cos\theta + \\ &+ (\gamma(0)\cos k_2 t - 2B(Pl)^{-1} k_2 \sin\varepsilon_0(0)\,\delta(0)\sin k_2 t)\sin\theta\end{aligned}$$

$$\begin{aligned}\gamma = -&(-k_1 g^{-1} V(0)\,\alpha(0)\sin k_1 t + \beta(0)\cos k_1 t)\sin\theta + \\ &+ (\gamma(0)\cos k_2 t - 2B(Pl)^{-1} k_2 \sin\varepsilon_0(0)\,\delta(0)\sin k_2 t)\cos\theta\end{aligned}$$

$$\delta = -\,{}^1/_2 PlB^{-1}\sin\varepsilon_0\,\{(V(0)\,\alpha(0)\cos k_1 t + g k_1^{-1}\beta(0)\sin k_1 t)\sin\theta + (g k_2^{-1}\gamma(0)\sin k_2 t + 2Bg(Pl)^{-1}\sin\varepsilon_0(0)\,\delta(0)\cos k_2 t)\cos\theta\} \qquad (6.5)\ \text{cont.}$$

These expressions can be simplified and from them we can obtain the formulas of Geckeler. When $\psi_0 = \frac{1}{2}\pi$ we have

$$\sin\theta = \mu\sin\omega t + O(\mu^3), \qquad \cos\theta = 1 + 0(\mu^2) \qquad (6.6)$$

Here $O(\mu^k)$ is the totality of all terms with μ^k and μ of higher degree. Consequently the first expression in (6.5) can be put in the form

$$\alpha = V^{-1}\{V(0)\,\alpha(0)\cos\nu t + g\nu^{-1}\beta(0)\sin\nu t + \mu\left(g\frac{1}{p}\gamma(0)\sin pt + \frac{2Bg\sin\varepsilon_0(0)}{Pl}\,\delta(0)\cos pt\right)\sin\omega t\} + O(\mu^2) \qquad (6.7)$$

If we set here $V \approx V(0) \approx Ru\cos\varphi$ and $\varepsilon_0 = \varphi$ and take into account (2.7) then with the accuracy up to the terms with μ^2 we shall have

$$\alpha = \alpha(0)\cos\nu t + \frac{\nu}{u\cos\varphi}\beta(0)\sin\nu t + \mu\left[\frac{\nu^2}{pu\cos\varphi}\gamma(0)\sin pt + \delta(0)\tan\varphi\cos pt\right]\sin\omega t \qquad (6.8)$$

Similarly we have

$$\beta = -\frac{u\cos\varphi}{\nu}\alpha(0)\sin\nu t + \beta(0)\cos\nu t + \mu\left[\nu(0)\cos pt - \frac{pu\sin\varphi}{\nu^2}\,\delta(0)\sin pt\right]\sin\omega t \qquad (6.9)$$

If the conditions of the maneuver are such that μ is a sufficiently small quantity, then we can take into account in (6.7) only the first term putting

$$\alpha = V^{-1}[V(0)\alpha(0)\cos\nu t + g\nu^{-1}\beta_0(0)\sin\nu t] \qquad (6.10)$$

This result can be obtained directly from the equations of Geckeler (1.2).

In the case of circulations from the northern course the final expressions for α, β, γ and δ will have similar form as in (6.5), but will be somewhat more complicated. In particular, the terms with $\cos\theta$ in the expressions for α and β will contain terms depending on $\gamma(0)$ and $\delta(0)$.

7. This analysis permits us to conclude that in investigating the free motion of a gyrocompass and its stability during a ship's circulation the principal criterion justifying the Geckeler simplification is the smallness of the parameter μ. When $\mu \ll 1$ the simplifications introduced by replacing the initial equations by the Geckeler equations do not change the essential picture of the phenomenon, and it leads in general to correct conclusions with respect to the expected accuracy of the course indication.

If the ship moves in such a way that the function $\Omega = \Omega(t)$ does not change its sign when the course is changed (this corresponds to single turns, or for example to a half circulation from the course 0°), then the method presented here cannot be applied, because the averaging of the coefficients of the system (2.13) was carried out through the period of a whole circulation. In such cases a very good accuracy can be obtained by using the method presented in [2]. Let us mention, that in the case of a half circulation from the northern course we can carry out the averaging through a semiperiod of a circulation.

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